BOOK REVIEWS

A FEW MATHEMATICAL TEXT-BOOKS

It is, perhaps, not expected that a review for American teachers should dwell at any length upon foreign works. If so, it is unfortunate, since it is to the continent of Europe that we must still look for the most inspiring, at least for most of the inspiring, text-books on mathematics. One of these is Dr. Böger's work on projective geometry, the latest addition to the Sammlung Schubert.[1] While the subject has as yet no place in the American secondary school, and not much of a standing in the college, it is becoming so prominent that high-school teachers can no longer afford to be ignorant of its scope and value. In English we have long been limited to the translation of Cremona, at least until the recent appearance of a translation of Reye's well-known work. A good text-book on the subject has, however, been much needed, and this want has been admirably supplied for German readers by Dr. Böger. He has, it is needless to say, followed Reye's presentation of von Staudt's theory, but has felt free to depart from it where there is a definite gain by doing so, as in the adoption of Thomas's definition of projective relations and in his treatment of imaginary elements. The distinguishing excellence of the book is its lucid style combined with its conciseness. Reye and Cremona are often prolix, as perhaps becomes the writers of treatises, but Böger projects the distinctive points of the theory upon the student's plane, even as he similarly treats the geometric points in space. The principle of duality plays its important part from the first, but the anharmonic ratio is not introduced until the reader is prepared for it through a considerable study of harmonic elements.

For one who cares for a scholarly but succinct presentation of the subject, this book will be found valuable.

Dr. Milne's latest arithmetic[2] is written in the style and with the purpose that characterizes his other works. The style is popular, and the purpose is to furnish an easy book that does not depart any more than necessary from the traditional. He gives the incorrect definitions of number, "Arabic" notation, multiplication, and similar terms that have become so familiar that most teachers have ceased to think about them. If we may judge by the number of problems under the various topics, Troy weight is more important than avoirdupois, and apothecaries' than Troy, all of which is quite discouraging to those who are endeavoring to eliminate from arithmetic the useless accumulations of the centuries. The exercises are, however, well graded, and not without interest, and herein lies the chief value of the book.

Mrs. Hornbrook's recent work[3] is much less traditional. It is true, however, that

[1] R. Böger, *Ebene Geometrie der Lage*, Leipzig, Göschensche Verlagshandlung, 1900, pp. 289, 5 marks.

[2] William J. Milne, *Intermediate Arithmetic*, American Book Co., 1900, pp. 219, price 30 cents.

[3] A. R. Hornbrook, *Grammar School Arithmetic*, American Book Co., 1900. pp. 416, price 65 cents.

she gives the same incorrect definition of "Arabic" notation, and that the unexplained rule too often finds place, as in the treatment of the least common multiple, percentage, the divisibility of numbers, and proportion. Intended for the grammar grades, it very properly introduces the rudiments of algebra and of metrical geometry, and does so in a skillful manner. It does not seem, however, that the value of the simple equation is sufficiently appreciated in the treatment of proportion. The book separates the subjects of ratio and proportion by about two hundred and fifty pages, an interesting innovation. The former topic precedes the treatment of fractions, which suggests a valuable, though often overworked, educational idea; this idea, however, seems to bear no fruit in the discussion. The book contains much that will appeal to good teachers as of value, especially in the questions at the beginnings of the various chapters. The weak features of the work are the unnecessary use of arbitrary rules and principles, and the monotonous arrangement of the discouragingly long lists of problems.

Somewhat more pretentious in title, but less valuable in fact, is Hall's little work on commercial arithmetic.[1] The book is written with the honest desire to present the most necessary parts of commercial arithmetic as it is found in America today. For this it is to be commended. The introductions to the chapters on stocks and bonds, banking, and exchange are valuable. But much of the rest of the book is open to such serious criticism as to make it certain that it will not be favorably received. The lower stratum of teachers will not use it because it attempts to be modern, and the upper stratum will not because of its faults. It is careless in its statements, as in its confusion of rate and rate per cent., in saying that the rate "has to be changed to a decimal before the problem can be solved," in asserting that a decimal fraction has to be transformed to become a rate, and in saying that 5 per cent. interest means "0.05 on every dollar," when it means 0.05 of any number of dollars. The product of two abstract numbers often appears in step form, as a concrete number, and there is a general carelessness in the matter of symbolism of this kind. Rules abound, often as formulas, and usually with so little explanation as to make their presence a source of danger. The book is divided into two parts, the first containing subjects not involving time, in which bonds are included, and the second those involving time, in which checks and sight drafts are explained, features that make one doubt the value of the division. Averaging of accounts takes more space than partial payments, a fact that may be justified on the score of difficulty, but certainly not from the importance of the subject.

With what pleasure does one emerge from the humdrum of the commonplace on getting hold of a book like Cantor's recent arithmetic of daily life.[2] It is with interest that we turn the leaves of a book on arithmetic by a professor in a German university, a man whose fame is world-wide in the line of the history of mathematics, and one whose name has always been connected, outside of Heidelberg, with the more advanced branches of the science. For many years, however, Professor Cantor has given, during the winter semester, a course of lectures upon political arithmetic for those interested in financial questions. The essence of these lectures is set forth in a

[1] GEORGE HALL, *The Common Sense of Commercial Arithmetic.* The Macmillan Company, 1901, pp. 187, price 60 cents.

[2] MORTIZ, CANTOR, *Politische Arithmetik oder die Arithmetik des täglichen Lebens;* Leipzig: Teubner, 1898, pp. 136, price 1.80 marks.

book so small as to be carried in one's pocket, but so scholarly as to become at once a standard. Within less than a hundred and fifty pages may be found historical and highly scientific discussions of simple and compound interest, probabilities and their applications in state lotteries (not uncommon abroad) and in the various forms of insurance, the computation of dividends, and other matters of finance as found in the real business life of this generation. It would do much good to those who still cry out for the disciplinary value of allegation, equation of payments, and compound proportion, to read a book of this kind and see the genuine discipline of genuine business. It would hasten the time when the problems of arithmetic would belong to the current generation, and this would be a novelty in education.

Professor Downey's *Higher Algebra*[1] is disappointing. A higher algebra today ought to mean more than Robinson's *University Algebra* of a generation ago, or Day's book of still an earlier generation. But while we find here one of the best pieces of book making that its publishers have recently produced, the matter is no particular improvement upon that of the books mentioned. Why should a "higher algebra" of today set forth a rule for every operation? Is modern pedagogy so wrong as to justify this reaction? Surely the world's great writers of today take the other view. At least it must be admitted that a rule requiring twenty-three lines for its statement can hardly be called "a *convenient* rule for this operation." (Horner's synthetic division.)

The author, too, lays himself open to serious criticism in his definitions. His *algebraic sum* seems to exclude the case of zero; his *algebraic difference* will hardly appeal to a student as lucid; the definition of *multiplication* excludes operations with irrational numbers; that of *degree* makes $a^{1/2}\, b^{1/2}$ of the first degree; that of *division* allows of a zero divisor; and that of *factor* $\sqrt{x}-1$ as a factor of $x-1$.

It must be admitted, too, that the demonstrations are very unsatisfactory. They might pass for a beginner's book, but not for a "higher algebra." The demonstration of the "rule" for subtraction is generally discarded today as begging the question, while that concerning the law of exponents in division is even more open to criticism. If these were isolated cases the book might be commended: but they are not, and their frequency justifies the remark that the book is disappointing.

A satisfactory presentation of the fundamental principles of non-Euclidean geometry, in succinct form, has long been desired by English readers. There have been any number of fugitive articles and brochures, but a convenient handbook to the subject has been wanting, and this want Professor Manning, of Brown University, has undertaken to fill.[2] On the whole this has been satisfactorily done. This does not mean that an exhaustive treatise has been attempted, or that any originality has been shown, save in the arrangement of matter; but a readable primer of the subject has been prepared, setting forth as much of the theory as most teachers will care for.

The arrangement is a convenient one. Recognizing that the denial of Euclid's postulate of parallels as *a priori* true leads to three possible geometries, the author seeks first to prove the fundamental propositions common to all three. He then proceeds to consider the two non-Euclidean geometries, the hyperbolic and the elliptic, in both two and three dimensional spaces. The work concludes with a brief historical note.

[1] JOHN F. DOWNEY, *Higher Algebra*, American Book Co., 1900, pp. 416, price $1.50.

[2] HENRY PARKER MANNING, *Non-Euclidean Geometry*. Ginn & Co., 1901. Pp. 95.

Our indebtedness to Dr. Manning may make it seem ungenerous to call attention to matters that will strike the reader unfavorably. Teachers are apt, however, to look upon books of this kind as models, and hence a word of caution is necessary. It is unfortunate that the author fails to distinguish between line and line segment, a distinction that would simplify the phraseology in several places. That "angles are measured by arcs" may be allowed for brevity in elementary text-books, but it hardly has place in a work like this. So the statement that "from any point without a line a perpendicular to the line *can be drawn*," followed by a proof that such a line merely exists, shakes one's confidence exactly as would a theorem stating that a regular heptagon can be inscribed in a circle, it being understood that the usual limitations of plane geometry are imposed. There is a similar statement regarding the drawing of only one such perpendicular. It would be difficult to understand what the author means by "the angles at the extremities of two equal perpendiculars," if he did not give a figure, and the lack of a definite order of letters in reading angles sometimes strikes the reader as unfortunate even where the figures are given. The rather antiquated forms, *hypothenuse* and *diedral*, are, of course, allowable; but the reason for the unusual factorial symbol (p. 59) is not so apparent.

The historical note is possibly extended enough for the purpose. The author has not consulted the standard works upon the subject, however, and his bibliography does not pretend to be complete.

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Experimental Psychology; A Manual of Laboratory Practice, Volume I, Part I, Student's Manual, price $1.60; Part II, Instructor's Manual, price $2.50. By EDWARD BRADFORD TITCHENER. New York: The Macmillan Company, 1901.

TEACHERS will be interested and pleased to find that Professor Titchener has set himself definitely to provide a text-book that shall be teachable. He has in fact taken infinite pains to aid instructors in the direction of a course in psychology, and to secure in students what he regards as the right attitude toward the course in all its details. The *Instructor's Manual*, of about 450 pages, is filled with references to psychological literature, descriptions of apparatus, details concerning the experiments; in fact, with such an outfit of knowledge concerning the course (reaching even to sixteen sets of examination questions) as a painstaking instructor might gather for himself in five or ten years' experience. The directions in the student's manual are detailed, clear, and precise. Any instructor who makes it the aim of a laboratory course to give students knowledge of the facts about our feelings of material objects and their relation to physical processes, and to train them in introspection and in the analysis of such feelings into mental elements, will undoubtedly find a use for Professor Titchener's Manual, in whole or in part.

But to say that he has done exceedingly well the thing he set out to do is not to agree that it is the best thing to do. Some, perhaps many, would propose a quite different aim and treatment for an elementary laboratory course in psychology. A majority of students of psychology are in professional schools for teachers. Is this sort of a course the best for them? The same question may be applied to students of